

Atomic entanglement mediated by a squeezed cavity field

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We consider the coherent state radiation field inside a micromaser cavity and study the entanglement mediated by it on a pair of two level atoms passing through the cavity one after the other. We then investigate the effects of squeezing of the cavity field on the atomic entanglement. We compute the entanglement of formation for the emerging mixed two-atom state and show that squeezing of the cavity radiation field can increase the atomic entanglement.

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I. Introduction

The most interesting idea associated with composite quantum systems is quantum entanglement. A pair of particles is said to be entangled in quantum mechanics if its state cannot be expressed as a product of the states of its individual constituents. This was first noted by Einstein, Podolsky and Rosen in 1935[1]. The preparation and manipulation of these entangled states that have nonclassical and nonlocal properties leads to a better understanding of basic quantum phenomena. For example, complex entangled states, such as the Greenberger, Horne, and Zeilinger triplets of particles[2] are used for tests of quantum nonlocality[3]. In addition to pedagogical aspects, entanglement has become a fundamental resource in quantum information processing[4] and there has been rapid development of this subject in recent years[5].

In recent years entanglement has been widely observed within the framework of atom-photon interactions such as in optical and microwave cavities[6]. An example that could be highlighted is the generation of a maximally entangled state between two modes in a single cavity using a Rydberg atom coherently interacting with each mode in turn[7]. The utility of entangled atomic qubits for quantum information processing has prompted several new methods for their generation[8]. In many of these schemes the transfer of entanglement between two different Hilbert spaces, i.e., from the photons to the atoms[9, 10], is involved. The properties of the radiation field involved govern the quantitative nature of atomic entanglement generated through such transfers.

The squeezed radiation field[11] has wide applications in many different arenas of quantum optics. The relation between squeezing and entanglement in general, is itself an interesting issue which has been discussed through many approaches in the literature[12]. Squeezing has been used as a resource in several protocols of generating and distilling entanglement, and in information

transfer[13]. In particular, it has been shown how atomic qubits can be entangled with the help of a squeezed radiation field using one or two optical cavities[14]. In this Letter we will study the effects of squeezing parameters of a squeezed radiation field inside a microwave cavity on the quantitative entanglement of atomic qubits passing through it.

The motivation for this work is to investigate the role of squeezing of the radiation field inside a cavity on the atomic entanglement mediated by it. We will focus on a micromaser system[10, 15] in which two-level Rydberg atoms are sent into the cavity at such a rate that the probability of two atoms being present there is negligibly small. Since the atoms do not interact directly with each other, the properties of the radiation field encountered by them bears crucially on the nature of atomic entanglement. We take the initial state of the two atoms as separate or product state, and the emergent two-qubit state is of a mixed entangled type[10]. The interaction between the atom and the field is governed by the Jaynes-Cummings model[16] which is experimentally realizable. We consider the cavity to be of a non-leaky type, i.e., $Q = \infty$, and the cavity-QED experiments are very close to such situations[6]. We quantify the two-atom entanglement by computing the entanglement of formation[17] and demonstrate how the entanglement can be increased by the squeezing of the radiation field, if the average cavity photon number is kept fixed.

We begin with a brief description of the basic framework. We consider a single mode cavity and two two-level atoms initially prepared in their upper excited states $|e_1\rangle$ and $|e_2\rangle$ which pass through the cavity one after the other. We first consider a coherent state field inside the cavity. A coherent state contains an indefinite number of photons and is a minimum uncertainty state[18] standing at the threshold of the classical-quantum limit. These states are parametrised by a single complex number α as follows:

$$|\alpha\rangle = \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (1)$$

A coherent state is an eigenstate of the annihilation operator a written as

$$a|\alpha\rangle = \alpha|\alpha\rangle \quad (2)$$

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and obeys a Poissonian distribution function in the photon number representation given by

$$P_n = \frac{e^{-\langle n \rangle} \langle n \rangle^n}{n!} \quad (3)$$

with the average photon number $\langle n \rangle = |\alpha|^2$. The distribution function P_n peaks at non-zero photon number, i.e., $n_{peak} \neq 0$.

The Janynes-Cummings interaction[16] leads to a tripartite joint state of the cavity field and the two atoms passing through it given by

$$\begin{aligned} |\Psi(t)\rangle_{a-a-f} = & \sum_n A_n [\cos^2(\sqrt{n+1}gt) |e_1, e_2, n\rangle \\ & + \cos(\sqrt{n+1}gt) \sin(\sqrt{n+1}gt) |e_1, g_2, n+1\rangle \\ & + \cos(\sqrt{n+2}gt) \sin(\sqrt{n+1}gt) |g_1, e_2, n+1\rangle \\ & + \sin(\sqrt{n+1}gt) \sin(\sqrt{n+2}gt) |g_1, g_2, n+2\rangle] \end{aligned} \quad (4)$$

where $P_n = |A_n|^2$ is the photon distribution function of the coherent state field. Since we are interested in calculating the entanglement of the joint two-atom state after the atoms emerge from the cavity, we consider the reduced density matrix $\rho(t)$ of the two-atom state given by

$$\rho(t) = \text{Tr}_{\text{field}}(|\Psi(t)\rangle_{a-a-f, a-a-f} \langle \Psi(t)|) \quad (5)$$

obtained after taking trace over the field variables. This state can be written in the matrix form in the basis of $|e_1\rangle$, $|e_2\rangle$, $|g_1\rangle$ and $|g_2\rangle$ states as

$$\rho_{a-a} = \begin{pmatrix} \gamma_1 & \gamma_7 & \gamma_8 & \gamma_6 \\ \gamma_7 & \gamma_2 & \gamma_4 & \gamma_9 \\ \gamma_8 & \gamma_4 & \gamma_3 & \gamma_{10} \\ \gamma_6 & \gamma_9 & \gamma_{10} & \gamma_5 \end{pmatrix}. \quad (6)$$

where

$$\begin{aligned} \gamma_1 &= \sum_n P_n \cos^4(\sqrt{n+1}gt), \\ \gamma_2 &= \sum_n P_n \cos^2(\sqrt{n+1}gt) \times \\ &\quad \sin^2(\sqrt{n+1}gt), \\ \gamma_3 &= \sum_n P_n \cos^2(\sqrt{n+2}gt) \times \\ &\quad \sin^2(\sqrt{n+1}gt), \\ \gamma_4 &= \sum_n P_n \sin^2(\sqrt{n+1}gt) \times \\ &\quad \cos(\sqrt{n+1}gt) \cos(\sqrt{n+2}gt), \\ \gamma_5 &= \sum_n P_n \sin^2(\sqrt{n+1}gt) \times \\ &\quad \sin^2(\sqrt{n+2}gt), \\ \gamma_6 &= \sum_n \sqrt{P_n P_{n-2}} \cos^2(\sqrt{n+1}gt) \times \\ &\quad \sin(\sqrt{n}gt) \sin(\sqrt{n-1}gt), \end{aligned}$$

$$\begin{aligned} \gamma_7 &= \sum_n \sqrt{P_n P_{n-1}} \cos^2(\sqrt{n+1}gt) \times \\ &\quad \cos(\sqrt{n}gt) \sin(\sqrt{n}gt), \\ \gamma_8 &= \sum_n \sqrt{P_n P_{n-1}} \cos^3(\sqrt{n+1}gt) \times \\ &\quad \sin(\sqrt{n}gt), \\ \gamma_9 &= \sum_n \sqrt{P_n P_{n-1}} \sin^2(\sqrt{n+1}gt) \times \\ &\quad \cos(\sqrt{n+1}gt) \sin(\sqrt{n}gt), \\ \gamma_{10} &= \sum_n \sqrt{P_n P_{n-1}} \sin^2(\sqrt{n+1}gt) \times \\ &\quad \cos(\sqrt{n+2}gt) \sin(\sqrt{n}gt). \end{aligned} \quad (7)$$

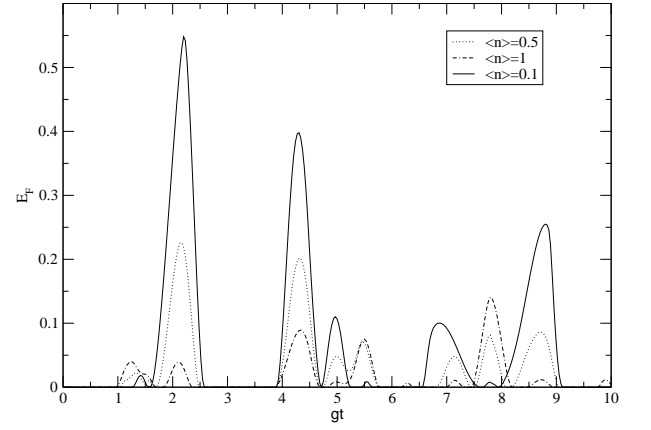


FIG. 1: Two-atom entanglement mediated by the coherent state cavity field at low average photon number is plotted versus gt .

We compute the entanglement of formation E_F for the state ρ_{a-a} , using the Hill-Wootters formula[17]

$$E_F(\rho) = h\left(\frac{1 + \sqrt{1 - C^2(\rho)}}{2}\right), \quad (8)$$

where C is called the concurrence defined as

$$C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (9)$$

where the λ_i are the eigenvalues of $\rho_{12}(\sigma_y \otimes \sigma_y) \rho_{12}^*(\sigma_y \otimes \sigma_y)$ in descending order, and

$$h(x) = -x \log_2 x - (1-x) \log_2 (1-x) \quad (10)$$

is the binary entropy function.

The entanglement of formation E_F is computed separately for low and high photon numbers as the two cases

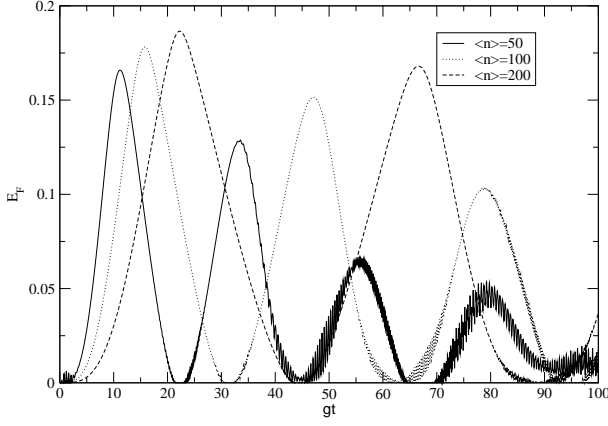


FIG. 2: Atom-atom entanglement mediated by coherent state cavity field at high average photon number is plotted versus gt .

have distinctive features for the coherent state field. E_f is plotted versus the Rabi angle gt (g is the atom-photon coupling constant of the Jaynes-Cummings interaction, and t is the time spent by the atom inside the cavity) for low average photon number $\langle n \rangle$ in Figure 1. The peaks of the entanglement of formation are reflective of the photon statistics that are typical in micro-maser dynamics[15]. We see that E_F falls off sharply as n increases. For small photon numbers, $n_{peak} \approx 0$ and hence, the evolution of E_F is similar to the case when a thermal field is inside the cavity[19]. For large $\langle n \rangle$, n_{peak} moves significantly to the right (Figure 2) and its influence is completely different compared to that for the low $\langle n \rangle$ case. Quantum effects which are predominant primarily when the photon number is low, help to increase the peak value of E_f . We note in Figure 2 that in general, E_F increases slightly with $\langle n \rangle$ with its time evolution being different for different $\langle n \rangle$. This is reflective of the collapse-revival characteristic in the Jaynes-Cummings model[15]. We further note that though E_F is higher for the low photon number category (Figure 1), this behaviour is reversed for the high photon number category (Figure 2). For high $\langle n \rangle$, the features of generated entanglement are thus significantly different from those in the case of the thermal field[19].

The above analysis sets the stage for the consideration of a squeezed radiation field inside the cavity. A class of minimum-uncertainty states are known as squeezed states. In general, a squeezed states have less noise in one quadrature than a coherent state. To satisfy the requirements of a minimum-uncertainty state the noise in the other quadrature is greater than that of a coherent state. Coherent states are a particular category of a more general class of minimum uncertainty states with equal noise in both quadratures. Our purpose here is to study what effect squeezing of the radiation field has on the entanglement of a pair of atoms passing through it. The

single mode field inside the cavity can be written as

$$E(t) = a_1 \cos \omega t + a_2 \sin \omega t \quad (11)$$

where $a_1 = (a + a^\dagger)/2$ and $a_2 = (a - a^\dagger)/2i$ are the two quadratures satisfying $[a_1, a_2] = i/2$. The variances $\Delta a_1 = \sqrt{\langle a_1^2 \rangle - \langle a_1 \rangle^2}$ and $\Delta a_2 = \sqrt{\langle a_2^2 \rangle - \langle a_2 \rangle^2}$ satisfy

$$\Delta a_1 \Delta a_2 \geq \frac{1}{4}. \quad (12)$$

The coherent state or the minimum uncertainty state given by Eqs.(1-3) satisfy the equality sign along with

$$\Delta a_1 = \Delta a_2 = \frac{1}{2}. \quad (13)$$

Further, either of Δa_1 or Δa_2 can be reduced below $\frac{1}{2}$ at the expense of the other such that Eq.(12) is satisfied, and radiation fields having such properties are called squeezed fields

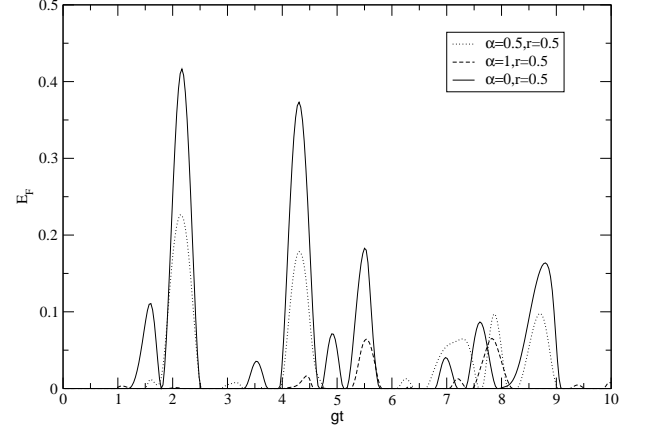


FIG. 3: Atom-atom entanglement of formation mediated by the squeezed field for different values of α is plotted versus gt for the low photon number case.

The photon distribution function of the squeezed radiation field can be represented as

$$P_n = \frac{1}{n! \mu} \left(\frac{\nu}{2\mu} \right)^n e^{-\beta^2 (\frac{\nu}{\mu} - 1)} |H_n(\frac{\beta}{\sqrt{2\mu\nu}})|^2, \quad (14)$$

where β is related to the coherent state amplitude α in Eq.(2) by $\beta = (\mu + \nu)\alpha$ for real α . μ and ν can be represented by the squeezing parameter r as $\mu = \cosh r$ and $\nu = \sinh r$. The average photon number can thus be written as

$$\langle n \rangle = |\alpha|^2 + \sinh^2 r. \quad (15)$$

In terms of the squeezing parameter, the variances of such fields are given by

$$\begin{aligned} \Delta a_1 &= \frac{1}{2} e^{-r}, \\ \Delta a_2 &= \frac{1}{2} e^r. \end{aligned} \quad (16)$$

Clearly, for $r = 0$, the statistics reduce to that for a coherent state given by Eq.(3). $r > 0$ gives rise to sub-Poissonian statistics, whereas $r < 0$ produces a super-Poissonian field.

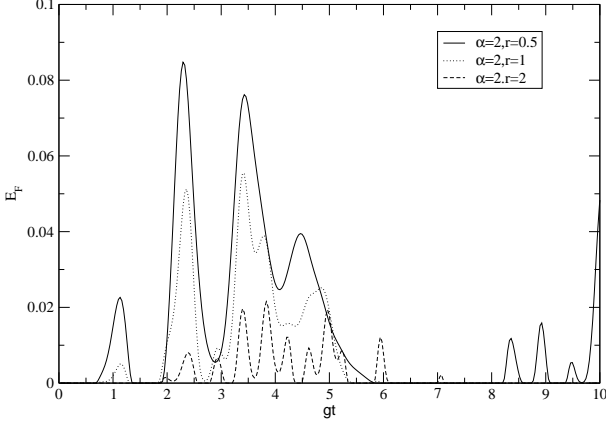


FIG. 4: E_F mediated by the squeezed field is plotted versus gt for different values of the squeezing parameter r corresponding to the low average photon number case.

As, in the previous case, we first obtain the reduced density matrix corresponding to the joint two-atom state after passing through a cavity with the squeezed field. The reduced density matrix has a similar form to that of the coherent state field given by Eq.(6), where γ^s are also of the same form as given in Eq.(7). The difference in this case arises from the different photon statistics P_n obtained from the squeezed field distribution function as given in Eq.(14).

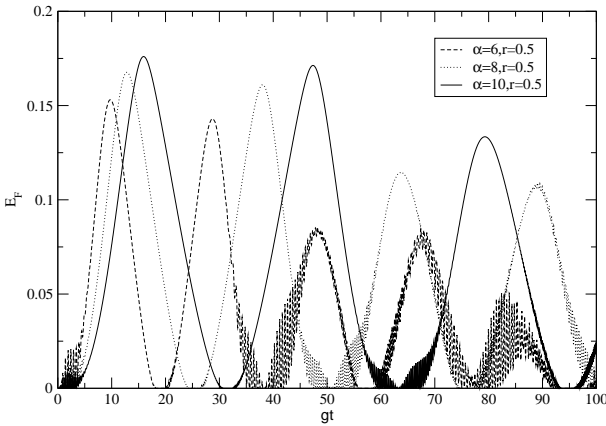


FIG. 5: E_F mediated by squeezed field for different values of α is plotted versus gt for the high average photon number case.

The effects of the photon statistics of the squeezed field on two-atom entanglement for low average photon number are displayed in the Figures 3 and 4, for varying α and r , respectively. We see that for low photon numbers, the time evolution of E_F is similar to that for a coherent field. The effect of the squeezing parameter r enters through $\langle n \rangle$ in Eq.(15). An increase in r increases $\langle n \rangle$ and thus E_F diminishes accordingly. It might appear from Figure 4 that squeezing of the radiation field is anti-correlated with the generated atomic entanglement, but what is actually reflected here is the decrease of E_F caused by the increase of the average photon number $\langle n \rangle$. We emphasize on this point since later (Figure 6) we will indeed see that by squeezing the field but holding $\langle n \rangle$ fixed, one can increase the atomic entanglement of formation. The situation for the high photon number case resembles that for the coherent state field. This is seen in Figure 5 where a larger value of α corresponds to a larger $\langle n \rangle$, and causes E_F to be slightly increased with increasing n or α .

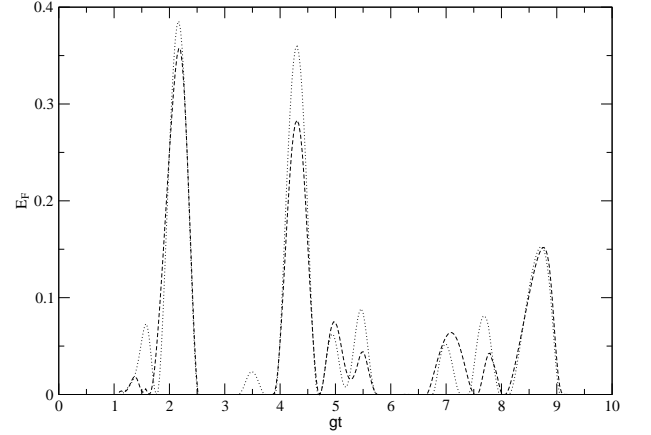


FIG. 6: Atom-atom entanglement mediated by (i) squeezed cavity field (dotted line) when $\langle n \rangle = 0.3$ and $r = 0.5$, and (ii) coherent state field (dashed line) when $\langle n \rangle = 0.3$, plotted vs gt .

The actual effect of squeezing of the cavity field is apparent by performing a comparative computation of E_F mediated by the coherent and squeezed fields for the same average photon number $\langle n \rangle$. In Figures 6 and 7 we plot the two-atom entanglement of formation E_F versus the Rabi angle gt separately for the coherent state and the squeezed state keeping the average cavity photon number fixed. In Figure 6 we see that for small $\langle n \rangle$, the dynamics of E_F are similar for both kinds of cavity fields. But the striking feature of Figure 6 is in the peaks of E_F for various values of gt . Note that E_F for the squeezed field (dotted line) is higher compared to the coherent state field (dashed line). Thus squeezing of the radiation field as represented by the non-vanishing value of the squeezing parameter r , leads to a notable increase

in the magnitude of atomic entanglement over the case the coherent state field ($r = 0$; no squeezing). This trend is also visible in the high photon number case (Figure 7), though not for all values of gt .

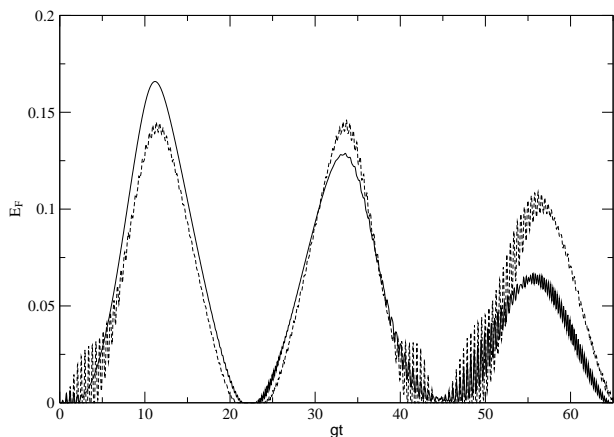


FIG. 7: Atom-atom entanglement mediated by (i) squeezed cavity field (dashed line) when $\langle n \rangle = 50$ and $r = 1$, and (ii) coherent state field (solid line) when $\langle n \rangle = 50$ when $\langle n \rangle = 50$, plotted vs gt .

To summarize, in this Letter we have considered a mi-

cromaser model where two spatially separated atoms are entangled via a cavity field. The entanglement between the two separate atoms builds up via atom-photon interactions inside the cavity, even though no single atom interacts directly with another. We have computed the two-atom entanglement as measured by the entanglement of formation E_F for the cases of the coherent state field and the squeezed radiation field inside the cavity. Our purpose has been to perform a quantitative study of the effects of squeezing of the bosonic radiation field on the mediation of the mixed state entanglement of two atomic qubits. Two distinct patterns of entanglement are seen to emerge for the cases corresponding to low and high average cavity photon numbers, respectively. In the former case the quantum nature of the radiation field plays a prominent role in enhancing atomic entanglement with the decrease of $\langle n \rangle$. The situation reverses for high $\langle n \rangle$ case where actually the increase of $\langle n \rangle$ leads to a slight increase of E_F . The key feature prominently observed for the low $\langle n \rangle$ case is that the two-atom entanglement can be increased with squeezing of the cavity field if the average cavity photon number is held fixed. Further interesting directions could be to study the impact of squeezed radiation on the “monogamous” character[20] of atomic entanglement, and also to investigate the possibility of generating maximally entangled mixed atomic qubits[21] using squeezing of the bosonic field as a resource.

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